A Universally Composable Scheme for Electronic Cash

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Abstract We propose a scheme for electronic cash based on symmetric primitives. The scheme is secure in the framework for universal composability assuming the existence of a symmetric CCA2-secure encryption scheme, a CMA-secure signature scheme, and a family of one-way, collision-free hash functions. In particular, the security proof is *not* in the random-oracle model. Due to its high efficiency, the scheme is well-suited for devices such as smart-cards and mobile phones. We also show how the proposed scheme can be used as a group signature scheme with one-time keys.

1 Introduction

1.1 Background and Previous Work

The concept of electronic cash, e-cash, was introduced by Chaum et al. [7], and several subsequent schemes have been proposed [2,9,21,20,18,16,15,4]. In an e-cash scheme there are three types of participants – the bank, merchants, and users. The users can withdraw coins from the bank and spend them at merchants. An e-cash scheme is *online* or *offline*. In the former case the bank is involved in every transaction, whereas in the second case payments can be performed without contacting the bank. Obviously offline schemes are preferable to online schemes. However, an electronic coin, being nothing but a string of numbers, can be copied and spent more than once, and in an offline scheme such double-spendings cannot be detected during the actual purchase. Rather than preventing double-spending, offline schemes are designed so that double-spenders are detected and identified.

Privacy is a crucial ingredient of e-cash schemes. It is desirable that merchants cannot learn the identity of the user, or even determine whether two payments were made by the same user or not. Many schemes also provide the same privacy towards the bank. However, anonymity also works in favor of criminals using the scheme for illegal activities protected by the privacy offered. To protect against such events some schemes offer the possibility for trusted third parties to trace a payment.

Most schemes require a merchant to deposit a coin after the purchase. A few schemes allow a coin to be transferred between users in several steps before it is

deposited at the bank [17,18]. Such schemes are said to have *transferable coins*. Another possible feature is divisability, i.e., that a coin may be spent only in part [18,16,15].

1.2 Group Signatures and E-cash Schemes

Group signatures were introduced by Chaum and van Heyst [8]. In a group signature scheme there is a *group manager* and *group members*. The group manager delegates the right to generate signatures to the group members, and also publishes a *group public key*. Members can sign messages, and a signature can be verified against the group public key, but only the group manager can open a signature to learn the identity of the signer. To anyone else the signature is anonymous.

Group signatures bear many resemblances to electronic cash. Group signatures are indistinguishable to anyone but the group manager in very much the same way payments are indistinguishable in anonymous e-cash schemes. One important difference is that there is no concept of double-spending for group signatures. See, e.g., [13] for an example of an e-cash scheme based on group signatures.

1.3 Our Contribution

All the above schemes involve trapdoor functions such as variants of ElGamal encryption or RSA groups. A real-life electronic cash scheme would probably be implemented on a portable device with low computational power such as a smart-card or a mobile phone. For such schemes it is important that the amount of computation is low, especially on the user side. The difference between zero, one or two exponentiations in the payment protocol is significant, whereas many schemes require tens, or in some cases hundreds, of exponentiations. The merchant terminal is more comparable to a low-end PC, but also in this case it is desirable to reduce the amount of computation to one or a few exponentiations.

Outline of Scheme. In this paper we propose a scheme which relies on symmetric primitives such as symmetric encryption, hash functions and pseudo-random functions. The only computations performed by the user during payment is evaluation of pseudo-random functions, and the merchant verifies a signature. It is commonly believed that there exist efficient algorithms for the primitives needed, e.g., AES and SHA-256. The scheme has been implemented on a mobile platform [22]. Our scheme builds along the lines of the scheme by Sander and Ta-Shma [20].

When a user $\mathcal{U}$ withdraws a coin, the bank encrypts the identity of $\mathcal{U}$. Then $\mathcal{U}$ uses a pseudo-random function to create a list of values and sends the hash values of the pseudo-random values to the bank. The coin, consisting of the encrypted identity and the hash values, is inserted as a leaf into a Merkle hash tree. After a certain amount of time, the bank builds the tree and publishes the

root. To spend a coin, the user reveals half of the preimages of the hash values together with a path from the coin up to a published root. The merchant verifies the correctness of the preimages, and verifies that the chain of hash values is valid.

If a user double-spends a coin, then she has revealed the preimage of more than half of the hash values. If this happens the bank decrypts the encrypted identity. From only the revealed preimages of a double-spent coin, it may be possible to successfully spend the coin a third time. In other words, a user double-spending a coin risks being held responsible for additional purchases. This gives additional incentive not to double-spend.

The anonymity of the scheme follows from the security of the encryption scheme, and unforgeability of coins follows since the hash function is collision-free.

As an additional feature of our scheme the payment protocol is non-interactive. In other words, the user produces a coin that can only be deposited by the designated merchant. This enables a user to prepare a coin for a certain merchant. In addition, anyone can verify that the coin has been prepared for that merchant. As an example, a parent can give a coin to their child which can be spent only at a certain store.

We show security for our scheme in the framework for universal composability (UC) [5]. We stress that our proof of security is in the plain model and not in the random-oracle model. We only assume that the encryption scheme is CCA2-secure, that the hash functions are one-way and collision-free, and that the pseudo-random functions are indistinguishable from random functions. We believe that the current scheme is the first scheme for electronic cash with a security proof in the UC-model and also the first practical scheme that does not use the random-oracle model for its security proof.

Previous e-cash schemes that are secure in the plain model include [20] and [12]. The former uses zero-knowledge proofs based on general methods, and the latter is a blind signature scheme using general methods for two-party computation from which an e-cash scheme may be built. Neither of these is practical.

Our scheme does not offer the same anonymity towards the bank as many other schemes. It is an interesting question whether a scheme that does not involve trapdoor functions can offer the anonymity towards the bank in the same strong sense as, e.g., [7]. For a more thorough discussion on this, see Appendix C.1.

Relations to Group Signatures. Although proposed as a scheme for electronic cash, our scheme has some similarities with group signatures. The bank has the ability to open a coin to extract the identity in the same way the group manager can open a signature. As a matter of fact, our scheme can be seen as a group signature scheme with one-time keys. This is discussed in further detail in Section 6.

Comparison to Sander-Ta-Shma. As in the scheme by Sander and Ta-Shma [20], in our scheme the bank builds a hash tree and the merchant uses the published root when verifying a coin. In their scheme a zero-knowledge protocol is used by the user to prove ownership of a preimage of a hash value and a path to some certified root. Focusing on efficiency, we avoid the zero-knowledge proof by letting the user reveal preimages of $\kappa/2$ out of κ hash values. The cost for this efficiency increase is that the bank always can identify the payer.

2 Notation and Definitions

2.1 Notation

String concatenation is denoted by $||$. For two integers a and b their concatenation $a||b$ is the number created by concatenating their binary representations, e.g., $a||b = 2^{k_b}a + b$, if b is a k_b-bit number. By $s \leftarrow_R S$ we mean that s is chosen independently and uniformly at random from the finite set S.

We define $[n] = \{1, 2, \ldots, n\}$. Let $\mathcal{I} = \{i_1, i_2, \ldots, i_k\}$, $i_j < i_{j+1}$, be a subset of $[n]$. For a list of values $\boldsymbol{v} = (v_1, v_2, \ldots, v_n)$ we define $\boldsymbol{v}_\mathcal{I} = (v_{i_1}, v_{i_2}, \ldots, v_{i_k})$. Let f be a function, $S = \{s_1, s_2, \ldots, s_m\}$ a set, and $\boldsymbol{v} = (v_1, v_2, \ldots, v_n)$ a vector. We define $f(S) = \{f(s_1), f(s_2), \ldots, f(s_m)\}$ and $f(\boldsymbol{v}) = (f(v_1), f(v_2), \ldots, f(v_n))$.

2.2 Basic Definitions

A function ε is *negligible* if $\varepsilon(\kappa) < 1/p(\kappa)$ for any polynomial $p(\kappa)$ and sufficiently large κ.

We define hash functions and pseudo-random functions as families of functions. The most common way to realize a family of functions is to define the function such that it depends on a key. A function is drawn from the family by generating a key.

We use hash functions that are collision-free, sometimes called collision-resistant, and one-way. Let $\mathcal{H}_\kappa$ be a family of hash functions that map values in $\{0,1\}^*$ to $\{0,1\}^\kappa$, and let $\mathcal{H} = \{\mathcal{H}_i\}_{i=1}^\infty$. Intuitively $\mathcal{H}$ is collision-free if it is infeasible to find two distinct inputs that hash to the same value and one-way if it is hard to compute a preimage of a random value for $H \leftarrow_R \mathcal{H}_\kappa$.

Let $\mathcal{R}_\kappa$ be a family of functions from $\{0,1\}^\kappa$ to $\{0,1\}^\kappa$, and let $\mathcal{R} = \{\mathcal{R}_i\}_{i=1}^\infty$. Let $\mathcal{U}_\kappa$ be the family of all functions from $\{0,1\}^\kappa$ to $\{0,1\}^\kappa$. Informally $\mathcal{R}$ is said to be *pseudo-random* if it is infeasible to distinguish a function from $\mathcal{R}_\kappa$ from a function from $\mathcal{U}_\kappa$.

A signature scheme $\mathcal{SS} = (\mathsf{Kg}, \mathsf{Sig}, \mathsf{Vf})$ is *correct* if $\mathsf{Vf}_{\mathsf{pk}}(m, \mathsf{Sig}_{\mathsf{sk}}(m)) = 1$ for $(\mathsf{pk}, \mathsf{sk})$ generated by Kg and any message m. $\mathcal{SS}$ is secure against chosen-message attacks, CMA-secure [10], if it is infeasible to produce valid message-signature pair for *any* message, even if the adversary has access to a signing oracle $\mathsf{Sig}_{\mathsf{sk}}(\cdot)$.

A symmetric encryption scheme $\mathcal{CS} = (\mathsf{Kg}, E, D)$ is secure against a chosen cipher-text attack, CCA2-secure, if it is infeasible to distinguish between encryptions of two messages of the adversary's choice, even if the adversary is

given access to an encryption oracle and a decryption oracle. This is a natural extension of CCA2-security for asymmetric encryption schemes [19].

Please see Appendix A for precise security definitions.

2.3 Merkle Trees and Hash Chains

Consider the task of proving that a value belongs to a set of certified values. One way to achieve this is to create a binary tree with the values as leaves by setting the value of every inner node to the hash value of the concatenation of the values of its two children and publish the root in a certified way. This is called a *Merkle tree* [14].

From a Merkle tree a *hash chain* from each leaf up to the root of the tree can be constructed. For each step the chain contains a value and an order bit which says whether the given value should be concatenated from the left or from the right.

An example of a Merkle tree is given in Figure 1. From the tree in the figure we can construct a hash chain from c_{121} up to the root as $(c_{121}, \rho, v_2, r, v_{11}, l, c_{122}, r)$. The values in the chain from c_{121} have been circled in the figure. Note that v_1 and v_{12} are not part of the chain, since these values are computed during verification.

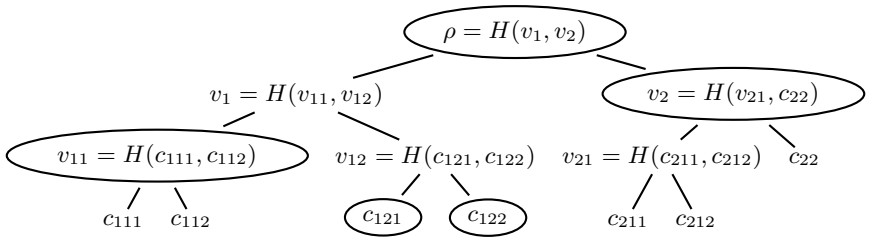

Figure 1. A Merkle tree with the values stored in the hash chain from c_{121} to the root marked.

Definition 1 (Hash chain). *A hash chain h of length d is a vector $h = (v, h_0, h_1, o_1, h_2, o_2, \ldots, h_{d-1}, o_{d-1})$ where $o_i \in \{l, r\}$. A hash chain is said to be valid under a hash function H if $h_0 = h'_0$, where $h'_{d-1} = v$ and*

$$h'_{i-1} = \begin{cases} H(h_i || h'_i) \text{ if } o_i = l \\ H(h'_i || h_i) \text{ if } o_i = r \end{cases}$$

for $i = d-1, d-2, \ldots, 1$. This is written $\mathsf{isvalid}_H(h) = 1$, or $\mathsf{isvalid}(h) = 1$ if it is clear from the context which hash function is used. We also define $\mathsf{root}(h) = h_0$ and $\mathsf{leaf}(h) = v$.

Once a Merkle tree has been built for a set of values and its root value has been published, constructing a hash chain for a value not in the set implies

finding a collision for the hash function. Since this is assumed to be infeasible, Merkle trees give a method of proving membership.

We define the randomized function $\mathsf{buildtree}_H(S)$ with input a set $S = \{s_1, s_2, \ldots, s_n\}$ to build and output a hash tree of depth $\lceil \log_2 n \rceil$ where the leaves have values $s_1, \ldots, s_n$ in random order. When n is a power of two, all leaves have equal depth $d-1$, and otherwise some leaves have depth $d-2$. The function $\mathsf{getchain}_T(s)$ returns the hash chain from the first leaf with value s to the root in the tree T and $\emptyset$ if no such leaf exists.

It is possible to join two Merkle trees into a new Merkle tree by creating a new root which has the two trees as children. Sander and Ta-Shma describe how the number of active roots can be reduced by joining the existing trees. In our scheme we do not join trees in this way, although the scheme could be modified to do so.

3 The Protocol

3.1 Security Parameters

Two security parameters, κ_1 and κ_2, are used in the protocol. The parameter κ_1 can be thought of as key length for the symmetric cipher, and κ_2 is the number of bits needed so that each merchant can be identified by a κ_2-bit number with $\kappa_2/2$ number of ones.

3.2 The Players

The players in the protocols are denoted $\mathcal{B}, P_1, \ldots, P_m$. To simplify the description we also write P_0 for $\mathcal{B}$. Except for the bank, any player may act as a customer, i.e., withdraw and spend coins, as well as a merchants, i.e., accept payments and deposit coins. We abuse notation and let P_i represent both the identity of the player and the Turing machine taking part in the protocol.

We let $\mathcal{I}$ be a public map from identities to $[\kappa_2]$ such that $\mathcal{I}(P_i)$ has cardinality $\kappa_2/2$ and $\mathcal{I}(P_i) \neq \mathcal{I}(P_j)$ for $P_i \neq P_j$. $\mathcal{I}$ can be thought of as a collision-free hash function which maps its input to $\{0,1\}^{\kappa_2}$ with the additional property that the number of 1's in the output is always exactly $\kappa_2/2$, although it is probably more practical to realize the map with a table. We define

$$\mathsf{span}_{\mathcal{I}}\left(\{P_{i_1}, P_{i_2}, \ldots, P_{i_k}\}\right) = \left\{ P \mid \mathcal{I}(P) \subseteq \bigcup_{j=1}^{k} \mathcal{I}(P_{i_j}) \right\} .$$

Given preimages of a coin corresponding to players $P_{i_1}, P_{i_2}, \ldots, P_{i_k}$ one can combine the preimages to spend the coin at any player in $\mathsf{span}_{\mathcal{I}}(\{P_{i_1}, P_{i_2}, \ldots, P_{i_k}\})$. It holds that $P \in \mathsf{span}_{\mathcal{I}}(S)$ if $P \in S$. Since $\mathcal{I}$ is injective $\mathsf{span}_{\mathcal{I}}(\{P\}) = \{P\}$.

4 The Ideal Functionality

4.1 Introduction

In this section we define the ideal functionality $\mathcal{F}_{\text{EC}}$ and discuss why it captures the properties of an e-cash scheme.

We use a model where the ideal functionality is linked to the players through a communication network $\mathcal{C}_{\mathcal{I}}$. The communication network forwards a message m from a player P as (P, m) to the ideal functionality. When $\mathcal{C}_{\mathcal{I}}$ receives (P, m) from the functionality, it forwards the message m to player P. Except for immediate functions, defined as a message from a player P immediately followed by a response to the same player P, the ideal adversary $\mathcal{S}$ is informed of when a message is sent, but not of the content. The ideal adversary is allowed to delay the delivery of such a message, but not change its content.

The functionality described here has only one non-immediate function – the withdrawal protocol.

The adversary is allowed to choose an arbitrary number of players to corrupt at start-up. For further discussion on this, see Appendix C.2. We do not allow the adversary to corrupt $\mathcal{B}$. It would be possible to give a functionality that allows the adversary to corrupt $\mathcal{B}$. In such a functionality even a corrupted $\mathcal{B}$ would not be able to "revoke" an issued coin. However, since this would make the functionality more complex, we describe $\mathcal{F}_{\text{EC}}$ for a trusted bank.

4.2 Informal Description

The ideal functionality $\mathcal{F}_{\text{EC}}$ for an e-cash scheme accepts the following messages.

- `KeyGen` to set up keys.
- `Issue Coin` to issue a coin to the designated user.
- `Tick` to build a new hash tree.
- `Prepare Coin` to mark a coin for spending at a certain merchant.
- `Verify Coin` to verify whether or not a coin can be spent at a certain merchant.
- `Open Coin` to let the bank extract the identity of the user the coin was issued to.
- `Check Doublespent` to check whether a coin has been spent more than once.

There is no separate message for depositing a coin at the bank. To deposit the merchant hands the coin to the bank, which runs the `Verify Coin` algorithm to check if the coin is valid.

4.3 Definition of the Ideal Functionality

The ideal functionality $\mathcal{F}_{\text{EC}}$ holds a counter t that is initialized to 0 and indexed sets C_i for coins that have been issued in period i. For convenience we let $C = \bigcup_i C_i$. For $e = (c, \cdot, k, \cdot) \in C$ we define $\mathsf{val}_H(e) = c||H(k_1)||H(k_2)||\cdots||H(k_{\kappa_2})$.

Table 1. The tables stored by the ideal functionality $\mathcal{F}_{\text{EC}}$.

Name	Content
C_i	$(c, P_i, \boldsymbol{k}, h)$, where c is a bit-string, P_i the coin-owner, $\boldsymbol{k}$ the coin-secret and h the hash chain.
T_{prepared}	Coins which are about to be spent.
T_{signed}	The certified roots.

The functionality holds a set of signed roots T_{signed} and a set of coins ready to be spent T_{prepared}. The sets $C_i, T_{\text{signed}}, T_{\text{prepared}}$ are initialized to $\emptyset$. The tables stored by the functionality are summarized in Table 1.

The functionality must be indistinguishable from the protocol, which implies that it must output data on the same format as the real protocol, and therefore it must depend on the implementation of the real protocol. This can be achieved by querying the ideal adversary for any such output, or the functionality can produce the output itself. In the former case, the ideal adversary needs to be tailor-made for a certain implementation of the protocol, whereas in the latter case the functionality must be parameterized by the implementation. We choose the second approach in this paper.

The functionality is parameterized by a symmetric encryption scheme $\mathcal{CS} = (\mathsf{Kg}, E, D)$, a signature scheme $\mathcal{SS} = (\mathsf{Kg}, \mathsf{Sig}, \mathsf{Vf})$, a family of pseudo-random functions $\mathcal{R}$ and collision-free, one-way hash functions H drawn from a family of hash functions $\mathcal{H}_{\kappa_1}$. To simplify the description we assume that $\mathcal{SS}$ is correct. Instead of parameterizing the functionality it is possible to give a non-parameterized description, where the functionality is given (a description of) the function families from $\mathcal{S}$ at startup. The definition of $\mathcal{F}_{\text{EC}}$ is given in Figure 2.

$\mathcal{F}_{\text{EC}}$ captures some specifics of the current scheme, such as the tree update function, the specific format of a coin, the weaker anonymity, a non-interactive payment protocol, and the possibility to transfer prepared coins to other users. Therefore a generic ideal functionality for electronic cash would differ from ours.

4.4 On the Ideal Functionality

In this section we discuss why $\mathcal{F}_{\text{EC}}$ captures the security requirements for electronic cash. The five messages `KeyGen`, `Issue Coin`, `Tick`, `Prepare Coin`, and `Check Doublespent` are all straight-forward. They manipulate tables, and use $H, \mathcal{R}, \mathcal{CS}, \mathcal{SS}$ only to produce output that has the format of a coin. When answering the `Open Coin` query, the functionality decrypts c if it is not found in the table. This is so since the CCA2-security of $\mathcal{CS}$ does not prevent $\mathcal{Z}$ from producing a valid cleartext-ciphertext pair and use it to determine whether it interacts with the functionality or the real protocol.

Since the most involved message is `Verify Coin`, we discuss it in more detail. As noted in [6], messages created by corrupted players or messages created with keys that do not originate from the protocol must be verified according to the real

Functionality 1 ($\mathcal{F}_{\mathrm{EC}}^{H,\mathcal{R},\mathcal{CS},\mathcal{SS}}$)**.** Until $(\mathcal{B}, \texttt{KeyGen})$ is received all messages except $(\mathcal{B}, \texttt{KeyGen})$ are ignored.

- Upon reception of $(P_i, \texttt{KeyGen})$ proceed as follows:
  1. If $P_i = \mathcal{B}$, set $\mathsf{key} \leftarrow \mathsf{Kg}(\kappa_1)$, $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{Kg}(\kappa_1)$, and return $(\mathcal{B}, \texttt{KeyGen}, \mathsf{pk})$.
  2. Else record P_i in the member list and draw U^i from the family $\mathcal{U}_{\kappa_1}$ and return $(P_i, \texttt{KeyGen})$.
- Upon reception of $(\mathcal{B}, \texttt{Issue Coin}, P_i)$, verify that P_i is in the member list. If not, return $(\mathcal{B}, \texttt{Not A Member})$ and quit. Set

$$c \leftarrow E_{\mathsf{key}}(0),\ k_j \leftarrow (U^i(c||j))_{j=1}^{\kappa_2},\ \boldsymbol{z} \leftarrow H(\boldsymbol{k})\ ,$$

  where $\boldsymbol{k} = (k_1, k_2, \ldots, k_{\kappa_2})$. Add $(c, P_j, \boldsymbol{k}, \emptyset)$ to C_t. Hand $(\mathcal{S}, \texttt{New Coin}, P_i)$ and $(P_i, \texttt{New Coin}, c, \boldsymbol{z})$ to $\mathcal{C}_{\mathcal{I}}$.
- Upon reception of $(\mathcal{B}, \texttt{Tick})$, set $T \leftarrow \mathsf{buildtree}_H(\mathsf{val}_H(C_t))$ and modify each $e = (c, P_i, \boldsymbol{k}, \emptyset) \in C_t$ into $(c, P_i, \boldsymbol{k}, \mathsf{getchain}_T(\mathsf{val}_H(e)))$. Set $\sigma \leftarrow \mathsf{Sig}_{\mathsf{sk}}(\mathsf{root}(T))$ and add $\mathsf{root}(T)$ to T_{signed}. Return $(\mathcal{B}, \texttt{Tick}, T, \sigma)$ to $\mathcal{C}_{\mathcal{I}}$. Set $t \leftarrow t + 1$.
- Upon reception of $(P_i, \texttt{Prepare Coin}, c, \boldsymbol{z}, P_j)$, find $\boldsymbol{k}$ such that $(c, P_i, \boldsymbol{k}, \cdot) \in C$. If no such $\boldsymbol{k}$ exists, then hand $\mathcal{C}_{\mathcal{I}}$ the message $(P_i, \texttt{Reject Prepare Coin}, c)$ and quit. Otherwise set $\tilde{\boldsymbol{k}} \leftarrow \boldsymbol{k}_{\mathcal{I}(P_j)}$, return $(P_i, \texttt{Prepared Coin}, c, \tilde{\boldsymbol{k}})$ to $\mathcal{C}_{\mathcal{I}}$ and store (c, P_j) in T_{prepared}.
- Upon reception of $(P_i, \texttt{Verify Coin}, c, \boldsymbol{z}, \tilde{\boldsymbol{k}}, P_j, h', \sigma, \mathsf{pk}')$, find $P_l, \boldsymbol{k}, h$ such that $(c, P_l, \boldsymbol{k}, h) \in C$. Return $(P_i, \texttt{Verify Coin}, c, P_j, \texttt{invalid})$ to $\mathcal{C}_{\mathcal{I}}$ if at least one of the following holds:
  1. No such entry exists.
  2. $\mathsf{pk} = \mathsf{pk}'$ and $\mathsf{root}(h) \notin T_{\mathrm{signed}}$.
  3. $\mathsf{Vf}_{\mathsf{pk}'}(\mathsf{root}(h), \sigma) = 0$.
  4. $h' \neq h$.
  5. P_l is not corrupted, and

$$(\tilde{\boldsymbol{k}} \neq \boldsymbol{k}_{\mathcal{I}(P_j)}) \vee (P_j \notin \mathsf{span}_{\mathcal{I}}(\{P \mid (c, P) \in T_{\mathrm{prepared}}\}))\ .$$

  6. P_l is corrupted and $H(\tilde{\boldsymbol{k}}) \neq \boldsymbol{z}_{\mathcal{I}(P_j)}$.

  Otherwise return $(P_i, \texttt{Verify Coin}, c, P_j, \texttt{valid})$ to $\mathcal{C}_{\mathcal{I}}$.
- Upon reception of $(\mathcal{B}, \texttt{Open Coin}, c)$, find a value $(c, P, \cdot, \cdot)$ in C. If no such entry exists, then set $P \leftarrow D_{\mathsf{sk}}(c)$. Return $(\mathcal{B}, \texttt{Open Coin}, c, P)$.
- Upon reception of $(P_l, \texttt{Check Doublespent}, c, \boldsymbol{z}, \tilde{\boldsymbol{k_1}}, \tilde{\boldsymbol{k_2}}, h, \sigma, P_{j_1}, P_{j_2})$ from $\mathcal{C}_{\mathcal{I}}$, execute $(\texttt{Verify Coin}, c, \boldsymbol{z}, \tilde{\boldsymbol{k_i}}, h, \sigma, P_{j_i})$ for $i = 1, 2$.
  1. If at least one execution returns $(\texttt{Verify Coin}, c, P_{j_i}, \texttt{invalid})$, then return $(P_l, \texttt{Check Doublespent}, c, \texttt{invalid})$ to $\mathcal{C}_{\mathcal{I}}$.
  2. If $P_{j_1} = P_{j_2}$ then return $(P_l, \texttt{Check Doublespent}, c, \texttt{no})$, otherwise return $(P_l, \texttt{Check Doublespent}, c, \texttt{yes})$ to $\mathcal{C}_{\mathcal{I}}$.

Figure 2. The definition of $\mathcal{F}_{\mathrm{EC}}^{H,\mathcal{R},\mathcal{CS},\mathcal{SS}}$.

protocol rather than rejected. Otherwise the environment $\mathcal{Z}$ could distinguish between the ideal functionality and the real protocol by creating a new pair of signature keys and sign a root with this new key pair. The same holds for a corrupted $\mathcal{U}$ which might leak its secret to $\mathcal{Z}$ to let $\mathcal{Z}$ prepare coins internally without interacting with the protocol.

When a coin is verified, Condition 1 says that it should be considered invalid if it has not been issued by $\mathcal{B}$. Condition 2 say that if the coin is being verified with the correct key public key, then it is valid only if $\mathcal{B}$ actually signed the root, and Condition 3 ensures a correct answer when the coin is verified with a different public key. Because of the correctness of $\mathcal{SS}$, Condition 3 always holds for $\mathsf{pk} = \mathsf{pk}'$ if the root has been signed. Condition 4 says that the correct path must be given. Condition 5 says that if the coin owner is not corrupted, the coin must have been prepared for the designated receiver P_j. (Recall that $\mathsf{span}_I(\{P\}) = \{P\}$.) Alternatively if the coin has been prepared more than once, then P_j must be in the span of the set of receivers. Condition 6 says that for a corrupt coin owner, the coin is accepted if the given preimages actually hash to the correct values.

Anonymity. In the ideal protocol, c is an encryption of 0, and thus the coin does not contain any information about the owner. The only information that is disclosed to the merchant is to which tree the coin belongs. The amount of information this contains depends on the size of the tree. The larger the tree, i.e., the longer the interval between `Tick` messages, the smaller the amount of information released to the merchant.

Fairness. By fairness we mean that if a player (or coalition of players) prepares $l + 1$ coins that pass `Verify Coin` after withdrawing only l coins, at least one withdrawn coin will be detected as double-spent. Since the only coins that can be successfully spent are the coins in the database C, and the only way to have a coin being added to the database is to engage in the withdrawal protocol, by the pigeon hole principle at least one coin has been prepared twice in this case. The implementation of the double-spending detection in the ideal functionality guarantees that the double-spender is revealed.

Non-frameability. A coalition of players should not be able to spend coins withdrawn by someone outside of the coalition. Since the `Prepare Coin` algorithm checks that it is called by the coin owner, this requirement is fulfilled.

Detection of double-spenders. A user that spends a coin at two different merchants will by construction have to produce two different sets of k_i's and will always be detected by the bank.

Since double-spending at a single merchant will not be detected by the bank, it is the responsibility of the merchant to detect such actions, holding a list of the coins spent at that merchant. However, a simple modification of the scheme

allows the merchant to remember only the coins spent the same day. To achieve this, we include the date in the computation of the index set. In the other words, rather than disclosing the list $\boldsymbol{k}_{\mathcal{I}(P_i)}$, the list $\boldsymbol{k}_{\mathcal{I}(P_i,date)}$ is disclosed.

Correctness. An e-cash scheme is correct if a coin withdrawn by an honest player always, or almost always, can be spent at an honest merchant and the merchant can deposit the coin at the bank. It is immediate from the construction that this property holds for the ideal functionality provided that the signature scheme $\mathcal{SS}$ is correct.

5 The Real Protocol

5.1 Definition of the Protocol

We give the definition of the protocol in the $\mathcal{F}_{\text{SIG}}$-hybrid model. The ideal signature functionality $\mathcal{F}_{\text{SIG}}$ [1,6] accepts messages $\texttt{KeyGen}, \texttt{Sign}, \texttt{Verify}$ to set up keys, sign a message, and verify a signature. We use the definition of $\mathcal{F}_{\text{SIG}}^{\mathcal{SS}}$ given in Figure 3, slightly modified from [6] in that the functionality is parameterized by the signature scheme, and that $\mathcal{SS}$ is assumed to be correct.

Functionality 2 ($\mathcal{F}_{\text{SIG}}^{\mathcal{SS}}$ [6]).

- Upon reception of $(\mathcal{B}, \texttt{KeyGen})$, set $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{Kg}$. Hand $(\mathcal{B}, \mathsf{pk})$ to $\mathcal{C}_{\mathcal{I}}$.
- Upon reception of $(\mathcal{B}, \texttt{Sign}, m)$, set $\sigma \leftarrow \mathsf{Sig}_{\mathsf{sk}}(m)$, store m, and hand $(\mathcal{B}, \texttt{Signature}, m, \sigma)$ to $\mathcal{C}_{\mathcal{I}}$.
- Upon reception of $(P_i, \texttt{Verify}, m, \sigma, \mathsf{pk}')$, set $f = 0$ if $\mathcal{B}$ is uncorrupted, m is not stored, and $\mathsf{pk} = \mathsf{pk}'$. Otherwise set $f = \mathsf{Vf}_{\mathsf{pk}'}(m, \sigma)$. Hand $(P_i, \texttt{Verify}, m, f)$ to $\mathcal{C}_{\mathcal{I}}$.

Figure 3. The definition of $\mathcal{F}_{\text{SIG}}^{\mathcal{SS}}$.

We are now ready to define the protocol $\pi_{\text{EC}}^{H,\mathcal{R},\mathcal{CS}}$.

Protocol 1 ($\pi_{\text{EC}}^{H,\mathcal{R},\mathcal{CS}}$).

- The bank $\mathcal{B}$ acts as follows:
  - Upon reception of $(\texttt{KeyGen})$, $\mathcal{B}$ creates and stores a symmetric key $\mathsf{key} \leftarrow \mathsf{Kg}(\kappa_1)$, requests pk from $\mathcal{F}_{\text{SIG}}$, sets $C \leftarrow \emptyset$ and returns $(\texttt{KeyGen}, \mathsf{pk})$.
  - Upon reception of $(\texttt{Issue Coin}, P_i)$, $\mathcal{B}$ initiates the following protocol with P_i:
    1. $\mathcal{B}$ computes $c \leftarrow E_{\mathsf{key}}(P_i)$ and sends $(\texttt{Withdrawal Request}, c)$ to P_i.
    2. P_i sets $k_j \leftarrow R^i(c||j), \boldsymbol{z} \leftarrow H(\boldsymbol{k})$. Then it outputs $(\texttt{New Coin}, c, \boldsymbol{z})$ and hands $(\texttt{Withdrawal Response}, c, \boldsymbol{z})$ to $\mathcal{B}$.
    3. $\mathcal{B}$ stores $(c||z_1||z_2||\ldots||z_{\kappa_2})$ in C.

- Upon reception of $(\texttt{Open Coin}, c)$, $\mathcal{B}$ returns $(\texttt{Open Coin}, c, D_{\mathsf{key}}(c))$.
- Upon reception of $(\texttt{Tick})$, $\mathcal{B}$ computes a new hash tree T from all stored values, i.e., sets $T = \mathsf{buildtree}_H(C)$. It acquires a signature σ on $\mathsf{root}(T)$ from $\mathcal{F}_{\mathrm{SIG}}$, sets $C = \emptyset$, and returns $(\texttt{Tick}, T, \sigma)$.

– A non-bank player P_i, i.e., $i > 0$, acts as below:

- Upon reception of $(\texttt{KeyGen})$, P_i creates and stores a pseudo-random function $R^i \leftarrow_R \mathcal{R}_{\kappa_1}$ and returns $(\texttt{KeyGen})$.
- Upon reception of $(\texttt{Prepare Coin}, c, \boldsymbol{z}, P_j)$, P_i sets $k_l \leftarrow R^i(c||l)$ for $l = 1, \ldots, \kappa_2$ and verifies that $\boldsymbol{z} = H(\boldsymbol{k})$. If this does not hold, it outputs the message $(\texttt{Reject Prepare Coin}, c)$ and quits. Otherwise it sets $\tilde{\boldsymbol{k}} = \boldsymbol{k}_{\mathcal{I}(P_j)}$ and outputs $(\texttt{Prepared Coin}, c, \tilde{\boldsymbol{k}})$.
- Upon reception of $(\texttt{Withdrawal Request})$, P_i acts as described above.

– In addition to the above, any player P_i, including the bank, acts as follows.

- Upon reception of $(\texttt{Verify Coin}, c, \boldsymbol{z}, \tilde{\boldsymbol{k}}, P_j, h, \sigma, \mathsf{pk})$, P_i proceeds as follows:
  1. P_i sends $(\texttt{Verify}, \mathsf{root}(h), \sigma, \mathsf{pk})$ to $\mathcal{F}_{\mathrm{SIG}}$. If $\mathcal{F}_{\mathrm{SIG}}$ returns 0, then P_i returns $(\texttt{Verify Coin}, c, P_j, \texttt{invalid})$ and quits.
  2. P_i verifies that $H(c, \boldsymbol{z}) = \mathsf{leaf}(h)$ and that $\mathsf{isvalid}_H(h) = 1$. If this is not the case, then P_i returns $(\texttt{Verify Coin}, c, P_j, \texttt{invalid})$ and quits.
  3. P_i verifies that $H(\tilde{\boldsymbol{k}}) = \boldsymbol{z}_{\mathcal{I}(P_j)}$. If this is not the case, then it returns $(\texttt{Verify Coin}, c, P_j, \texttt{invalid})$ and quits.

  P_i returns $(\texttt{Verify Coin}, c, P_j, \texttt{valid})$.
- Upon reception of $(\texttt{Check Doublespent}, c, \boldsymbol{z}, \tilde{\boldsymbol{k_1}}, \tilde{\boldsymbol{k_2}}, h, \sigma, P_{j_1}, P_{j_2})$, P_i executes $(\texttt{Verify Coin}, c, \boldsymbol{z}, \tilde{\boldsymbol{k_i}}, h, \sigma, P_{j_i})$ for $i = 1, 2$.
  1. If at least one execution returns $(\texttt{Verify Coin}, c, P_{j_i}, \texttt{invalid})$, then P_i returns $(\texttt{Check Doublespent}, c, \texttt{invalid})$ and quits.
  2. If $P_{j_1} = P_{j_2}$ then P_i returns $(\texttt{Check Doublespent}, c, \texttt{no})$, otherwise it returns $(\texttt{Check Doublespent}, c, \texttt{yes})$.

5.2 On the Real Protocol

The protocol relies on the existence of an ideal signature functionality. Such a functionality can be implemented with a CMA-secure signature scheme [1,6]. It is possible that a merchant will verify several coins from the same tree. In these cases the merchant can save time by only verifying the signature once.

The scheme relies on the roots being constructed after a certain amount of time, and therefore coins may not be immediately usable. The scheme can also be used without this delay by constructing a tree of size one for each coin issued and returning the signature to the user immediately. This increases coin size since there is a separate signature for each coin, but does not increase the amount of computation the user has to perform. This modification also eliminates linkability issues when coins with same owner are placed in the same tree.

5.3 Security of the Real Protocol

In Appendix B we prove the following theorem:

Theorem 1. *The protocol* $\pi_{\mathrm{EC}}^{H,\mathcal{R},\mathcal{CS}}$ *securely realizes* $\mathcal{F}_{\mathrm{EC}}^{H,\mathcal{R},\mathcal{CS},\mathcal{SS}}$ *in the* $\mathcal{F}_{\mathrm{SIG}}^{\mathcal{SS}}$*-hybrid model if* H *is drawn from a collection* $\mathcal{H}$ *of one-way collision-free hash functions,* $\mathcal{R}$ *is a collection of pseudo-random functions, and* $\mathcal{CS}$ *is a* CCA2*-secure encryption scheme.*

6 Comparison to Group Signatures

Our scheme is in some ways similar to group signatures. A coin can be viewed as a signature on the identity of the merchant. Signatures by different users are indistinguishable to merchants, but not to the bank. This corresponds to a group signatures scheme where the bank acts as group manager.

A user can only sign once for every coin she withdraws. For electronic cash this is a fundamental property, but it differs, of course, from ordinary group signatures. Also when used a group signatures scheme, there is no exculpability against the group manager. In other words the group manager can frame a group member.

Some group signature schemes offer revocation. When converting our scheme into a group-signature like scheme this can be achieved by publishing the coins issued to the revoked player. When verifying a signature, the verifier first checks the coin against the revocation list.

7 Conclusions

We have given a scheme for electronic cash that is based on symmetric primitives. The construction is efficient, and can be implemented on mobile phones or smartcards without a cryptographic co-processor. We have also showed how to convert the scheme into a group signature scheme with one-time keys.

8 Acknowledgments

I would like to thank Johan Håstad and Gustav Hast for discussions about the construction. I would also like to thank Douglas Wikström for suggesting improvements in the ideal functionality.

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A Definitions

We use hash functions that are collision-free, sometimes called collision-resistant, and one-way. Intuitively a hash function is collision-free if it is infeasible to find two distinct inputs that hash to the same value and one-way if it is hard to compute a preimage of a random value. The following two experiments define these two properties. Let H_κ be a family of hash functions that map values in $\{0,1\}^*$ to $\{0,1\}^\kappa$, and let $\mathcal{H} = \{\mathcal{H}_i\}_{i=1}^{\infty}$.

Experiment 1 (Collision-free function, $\mathbf{Exp}^{\mathsf{col}}_{\mathcal{H},A}(\kappa)$).

$H \leftarrow_R \mathcal{H}_\kappa$
$(m_0, m_1) \leftarrow A(\mathsf{guess}, H)$
if $(m_0 \neq m_1) \wedge (H(m_0) = H(m_1))$ **then**
  return 1
else
  return 0
end if

Experiment 2 (One-way function, $\mathbf{Exp}^{\mathsf{ow}}_{\mathcal{H},A}(\kappa)$).

$H \leftarrow_R \mathcal{H}_\kappa$
$x \leftarrow_R \mathrm{Im}(H)$
$y = A(\mathsf{guess}, H, x)$
if $H(y) = x$ **then**
  return 1
else
  return 0
end if

The advantage of an adversary A in the experiments above is defined as $\mathbf{Adv}^{\mathsf{col}}_{\mathcal{H},A}(\kappa) = \Pr[\mathbf{Exp}^{\mathsf{col}}_{\mathcal{H},A}(\kappa) = 1]$ and $\mathbf{Adv}^{\mathsf{ow}}_{\mathcal{H},A}(\kappa) = \Pr[\mathbf{Exp}^{\mathsf{ow}}_{\mathcal{H},A}(\kappa) = 1]$, respectively. $\mathcal{H}$ is *collision-free* and *one-way* if $\mathbf{Adv}^{\mathsf{col}}_{\mathcal{H},A}(\kappa)$ and $\mathbf{Adv}^{\mathsf{ow}}_{\mathcal{H},A}(\kappa)$, respectively, are negligible for any polynomial-time adversary A.

Let $\mathcal{R}_\kappa$ be a family of functions from $\{0,1\}^\kappa$ to $\{0,1\}^\kappa$, and let $\mathcal{R} = \{\mathcal{R}_i\}_{i=1}^\infty$. Let $\mathcal{U}_\kappa$ be the family of all functions from $\{0,1\}^\kappa$ to $\{0,1\}^\kappa$. Informally $\mathcal{R}$ is said to be pseudo-random if is infeasible to distinguish a function from $\mathcal{R}$ from a random function. The following experiment is used to formalize this.

Experiment 3 (Pseudo-random, $\mathbf{Exp}^{\mathsf{prf}-b}_{\mathcal{R},A}(\kappa)$).
if $b = 0$ **then**
  $f \leftarrow_R \mathcal{R}_\kappa$
else
  $f \leftarrow_R \mathcal{U}_\kappa$
end if
return $A^{f(\cdot)}(\mathsf{guess})$

The advantage of an adversary A is

$$\mathbf{Adv}^{\mathsf{prf}}_{\mathcal{R},A}(\kappa) = |\Pr[\mathbf{Exp}^{\mathsf{prf}-0}_{\mathcal{R},A}(\kappa) = 1] - \Pr[\mathbf{Exp}^{\mathsf{prf}-1}_{\mathcal{R},A}(\kappa) = 1]| \ .$$

The ensemble $\mathcal{R}$ is *pseudo-random* if $\mathbf{Adv}^{\mathsf{prf}}_{\mathcal{R},A}(\kappa)$ is negligible for any polynomial-time adversary A.

A signature scheme $\mathcal{SS} = (\mathsf{Kg}, \mathsf{Sig}, \mathsf{Vf})$ is secure against chosen-message attacks, CMA-secure [10], if it is infeasible to produce valid message-signature pair for *any* message, even if the adversary has access to a signing oracle $\mathsf{Sig}_{\mathsf{sk}}(\cdot)$. Formally we use the following experiment for the definition

Experiment 4 (CMA, $\mathbf{Exp}^{\mathsf{cma}}_{\mathcal{SS},A}(\kappa)$).
$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{Kg}(\kappa)$
$(m, \sigma) \leftarrow A^{\mathsf{Sig}_{\mathsf{sk}}(\cdot)}(\mathsf{pk})$
return $\mathsf{Vf}_{\mathsf{pk}}(m, s) = 1$

The advantage of the adversary is defined as

$$\mathbf{Adv}^{\mathsf{cma}}_{\mathcal{SS},A}(\kappa) = \Pr[\mathbf{Exp}^{\mathsf{cma}}_{\mathcal{SS},A}(\kappa) = 1] \ .$$

A signature scheme $\mathcal{SS}$ is CMA-secure if $\mathbf{Adv}^{\mathsf{cma}}_{\mathcal{SS},A}(\kappa)$ is negligible for all polynomial-time adversaries A.

Given a symmetric encryption scheme $\mathcal{CS} = (\mathsf{Kg}, E, D)$ the following experiment is used to define chosen cipher-text security (CCA2) [19]. Here $\mathcal{Q}(f(\cdot))$ denotes the set of questions answered by the oracle for $f(\cdot)$.

Experiment 5 (CCA2, $\mathbf{Exp}^{\mathsf{cca2}-b}_{\mathcal{CS},A}(\kappa)$).
$(\mathsf{sk}) \leftarrow \mathsf{Kg}(\kappa)$
$(m_0, m_1, \mathrm{state}) \leftarrow A^{E_{\mathsf{sk}}(\cdot), D_{\mathsf{sk}}(\cdot)}(\mathsf{choose})$
$c \leftarrow E_{\mathsf{sk}}(m_b)$
$d \leftarrow A^{E_{\mathsf{sk}}(\cdot), D_{\mathsf{sk}}(\cdot)}(\mathsf{guess}, c, \mathrm{state})$
if $c \in \mathcal{Q}(D_{\mathsf{sk}}(\cdot))$ **then**
  return 0
else

return d
end if

Let the advantage of A be

$$\mathbf{Adv}^{\mathsf{cca2}}_{\mathcal{CS},A}(\kappa) = |\Pr[\mathbf{Exp}^{\mathsf{cca2}-0}_{\mathcal{CS},A}(\kappa) = 1] - \Pr[\mathbf{Exp}^{\mathsf{cca2}-1}_{\mathcal{CS},A}(\kappa) = 1]| \ .$$

The encryption scheme $\mathcal{CS}$ is CCA2-secure if $\mathbf{Adv}^{\mathsf{cca2}}_{\mathcal{CS},A}(\kappa)$ is negligible for any polynomial-time adversary A.

B Proof of Theorem 1

Proof. We divide the proof into subsections. First we define the simulator, then we define the hybrids used and finally we describe how to break one of the assumptions if an environment can distinguish between the ideal functionality and the protocol.

Description of the Simulator. The simulator works as follows: For each player P_i that the real-world adversary $\mathcal{A}$ corrupts, the ideal adversary $\mathcal{S}$ corrupts the corresponding dummy player $\tilde{P}_i$. When a corrupted dummy player $\tilde{P}_i$ receives a message m from $\mathcal{Z}$, the simulator $\mathcal{S}$ lets $\mathcal{Z}'$ send m to P_i. When a corrupted P_i outputs a message m to $\mathcal{Z}'$, then $\mathcal{S}$ instructs the corrupted $\tilde{P}_i$ to output m to $\mathcal{Z}$. This corresponds to P_i being linked directly to $\mathcal{Z}$.

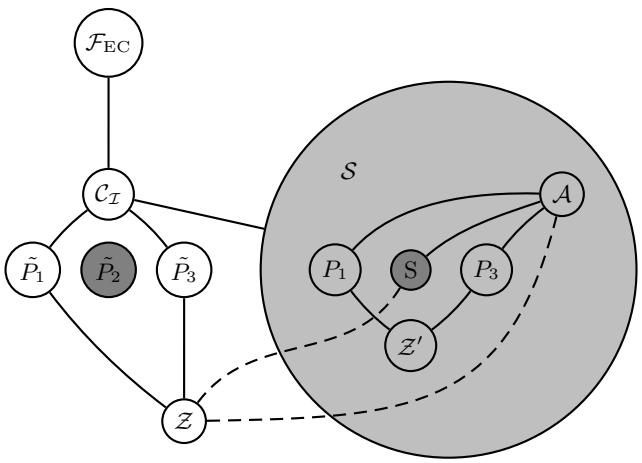

Figure 4. The simulator for a protocol with three players where P_2 is corrupted. The dashed edges represent simulated connections.

The simulated real-world adversary $\mathcal{A}$ is connected to $\mathcal{Z}$, i.e., when $\mathcal{Z}$ sends m to $\mathcal{S}$, $\mathcal{Z}'$ hands m to $\mathcal{A}$, and when $\mathcal{A}$ outputs m to $\mathcal{Z}'$, $\mathcal{S}$ hands m to $\mathcal{Z}$. All non-corrupted players are simulated honestly. The corrupted players run according to their respective protocols.

If $\mathcal{S}$ receives the message $(\texttt{New Coin}, P_i)$ from $\mathcal{F}_{\text{EC}}$, then it instructs $\mathcal{Z}'$ to send $(\texttt{Issue Coin}, P_i)$ to $\mathcal{B}$. All other functions are local and need not be simulated for $\mathcal{A}$.

Building the Hybrids. Now assume there exists an environment $\mathcal{Z}$ that can distinguish between an execution of the ideal protocol and an execution of the real protocol for any ideal adversary $\mathcal{S}$. Then it can distinguish between the two for the simulator described above. We will create a chain of protocols $\pi_0, \ldots, \pi_t$ such that π_0 is the ideal protocol and π_t the real protocol. We construct such a chain of polynomially many intermediate steps. If $\mathcal{Z}$ can distinguish between the ideal protocol and the real protocol, then there must exist an i such that $\mathcal{Z}$ can distinguish between π_i and π_{i+1}. We now build the chain and describe how $\mathcal{Z}$ can be turned into a machine that solves one of the problems assumed to be hard. To simplify the description we build the hybrid chains as several subchains called π^0, π^1 etc. We assume all chains have the same length m. Should a chain π^r as described below have length $m' < m$ we can always "pad" by letting $\pi^r_i = \pi^r_{m'}$ for $i = m' + 1, \ldots, m$. The chain built in this way is shown in Figure 5.

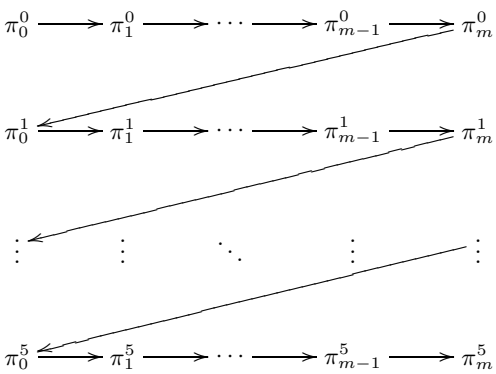

Figure 5. The chain of hybrid protocols.

We let π_0^0 be the ideal protocol. We then let π_i^0 be the same protocol with the difference that up to the ith coin issued we set c to be an encryption of the identity, $c = E_{\mathsf{key}}(P_i)$, rather than $c = E_{\mathsf{key}}(0)$.

We define π_0^1 to be π_m^0. Define π_i^1 to be π_0^1 with the modification that for the first i calls to `Open Coin` actually decrypt c according to the real protocol instead of looking up the answer in the table.

Let $\pi_0^2 = \pi_m^1$. Define π_i^2 to be π_0^2 with the difference for player $P_1, \ldots, P_i$ a pseudo-random function is used to generate k_i instead of the random function U^i.

Let $\pi_0^3 = \pi_m^2$. Define π_i^3 to be π_0^3 with the modification that up to the ith time `Check Doublespent` is called `yes` is returned if more than $\kappa_2/2$ hash values have been opened rather than using the table.

The protocol π^4 is defined as π^3 with the difference that the ith call to `Verify Coin` checks whether the path p is valid rather than checks that the coin exists in the table.

Finally the hybrid π^5 is defined to be π^4 with the modification that for π_i^5 the i first times the `Verify Coin` algorithm is called it is checked that $\kappa_2/2$ values k_i hash to z_i rather then checking them against the tables T_{prepared} and T_{signed}.

Breaking the Assumption. Assume $\mathcal{Z}$ can distinguish between π_i^0 and π_{i+1}^0 for some i. We show how to use $\mathcal{Z}$ to build an algorithm A that breaks the CCA2-security of $\mathcal{CS}$. Participating in Experiment 5 A has access to an encryption and a decryption oracle. No symmetric key is generated, but instead the encryption oracle is used to correctly form the first i coins, and the decryption oracle is used to open coins. When the $(i+1)$st coin is about to be created with identity P_j, A asks the challenge oracle to encrypt either 0 or P_j. All subsequent coins are created according to the ideal functionality.

Note that if the challenge oracle encrypts 0, the protocol executed is π_i^0, and if it encrypts P_j, the protocol is π_{i+1}^0. Since $\mathcal{Z}$ is able to distinguish between π_i^0 and π_{i+1}^0 with non-negligible probability it will break the CCA2-security of $\mathcal{CS}$.

The protocols in hybrid chain π^1 answer the `Open Coin` query identically, and thus $\mathcal{Z}$ cannot distinguish between them.

Assume $\mathcal{Z}$ can distinguish between π_i^2 and π_{i+1}^2 for some i. We show how to use $\mathcal{Z}$ to build an algorithm A that is able to distinguish between a pseudo-random function R and a random function U, thus contradicting the assumption that R is pseudo-random. A is given oracle access to a function f. For players $P_1, \ldots, P_i$ a pseudo-random function is used as in the real protocol. When coins are created for player P_{i+1}, the oracle for f is used to generate k_j, and for players P_l, $l > i+1$, the random function is used as in the ideal protocol.

If f is drawn from $\mathcal{U}_{\kappa_1}$, then the protocol described is π_i^2, and if f is drawn from $\mathcal{R}_{\kappa_1}$, then the protocol is π_{i+1}^2. Thus if $\mathcal{Z}$ is able to distinguish between π_i^2 and π_{i+1}^2 with non-negligible probability p, then A can distinguish between pseudo-random functions and random functions with the same probability p.

By construction instances of π^3 are indistinguishable, since no two $P_i \neq P_j$ have the same associated index set.

Assume $\mathcal{Z}$ distinguishes between π_i^4 and π_{i+1}^4. This means that $\mathcal{Z}$ with non-negligible probability has created a coin $(c, \boldsymbol{z})$ and corresponding path h such that

- $\mathsf{root}(h)$ was signed by $\mathcal{B}$.
- h was not in the original tree created by $\mathcal{B}$.

From this we can construct an algorithm A that given a key for the hash function finds a collision, contradicting the assumption that $\mathcal{H}$ is collision-free.

Assume $\mathcal{Z}$ distinguishes between π_i^5 and π_{i+1}^5 with non-negligible probability p_1. In that case, $\mathcal{Z}$ has succeeded to provide `Verify Coin` with $c, \boldsymbol{z}$ and values k_i such that either

- no entry c, P_j exists in T_{prepared}, or
- the values k_i do not match the values in the database.

and $H(k_i) = z_i$.

In the first case, the corresponding `Prepare Coin` has not been executed, and thus the values of k_i have not been revealed. We can now produce an algorithm A that given y computes $x \in H^{-1}(y)$, contradicting the assumption that H is hard to invert. Let us assume that coin verified in the $(i+1)$st call to `Verify Coin` was issued to P in the jth call to `Issue Coin` with non-negligible probability p_2. A runs the protocol honestly except that $z_l = y$, where l is the smallest index in the index set $\mathcal{I}(P)$. Since $\mathcal{Z}$ is able to make `Verify Coin` accept, it must have provided x such that $H(x) = y$. A then outputs x. Under the assumptions A succeeds with probability at least $p_1 p_2$.

In the second case, we can construct an algorithm that finds a collision in the hash function.

We have now shown that if $\mathcal{Z}$ can distinguish between π_{EC} and $\mathcal{F}_{\text{EC}}$, it can be used either to break the CCA2-security of $\mathcal{CS}$, to break the CMA-security of $\mathcal{SS}$, to distinguish between $\mathcal{R}$ and random functions, to find a collision in H, or to compute $H^{-1}(x)$ for a random x. Since this is assumed to be hard, the proof is concluded.

C Additional Notes

C.1 Can We Do Better?

As seen above, the proposed scheme lacks some of the properties one could ask from an e-cash scheme. Also, when used as a group signature scheme, it it does not have all the properties one could wish for. The reason for this is that we base the scheme on symmetric rather than asymmetric primitives. A natural question to ask is whether one could do better using only symmetric primitives.

It is known [3] that the existence of a group signature scheme implies existence of a CCA2-secure public-key encryption scheme. Impagliazzo and Rudich [11] show that it is unlikely that a public-key encryption scheme can be based only on the assumption of the existence of one-way functions where the function is used as a black box. Actually, the existence of such a construction would give a proof that $\mathbf{P} \neq \mathbf{NP}$. Therefore a construction of a group signature scheme from black-box access to symmetric primitives is likely to be extremely involved.

For electronic cash the situation is less clear. If double-spenders are detected only by the bank and not by anyone else (including the merchant), then it is possible to reduce CCA2-encryption to electronic cash in the same way as for group signatures. The same holds if there is a trusted third party that can identify coin owners. However, we are not aware of such a reduction from e-cash in general.

These restrictions only hold when there is only black-box access to symmetric primitives. When one is allowed access to the circuit computing a one-way functions, it is possible to, e.g., prove in zero-knowledge the knowledge of a preimage of a value under a hash function. Although polynomial, such proofs would most likely be highly inefficient.

C.2 On Adaptive Security

The security proof assumes that the adversary corrupts players in a non-adaptive way. An adversary that is allowed to corrupt adaptively would be able to distinguish between the real protocol and the ideal functionality. In the ideal functionality, the preimages a user $\mathcal{U}$ uses are random numbers, whereas in the real protocols they are pseudo-random numbers. When corrupting $\mathcal{U}$, the adversary expects to receive a key for the pseudo-random function that matches the preimages. In the ideal protocol the probability that such a key even exists is negligible.

We can modify the real protocol to solve the problem. If $\mathcal{U}$ instead of generating the preimages from a pseudo-random function generates random numbers, the above scenario does not apply. The drawback is that the amount of data that $\mathcal{U}$ needs to store increases.

C.3 Coin Size and External Databases

As the reader may have noted, the hash chains do not contain any sensitive information. Therefore they can be stored in public databases rather than by the user. This gives a way to reduce the size of the coins by storing only an index of the hash root together with the path in the tree as a $\{0,1\}$ string. The merchant can retrieve the hash values and $(c, \boldsymbol{z})$ from a public database.

Since the databases do not need to be authenticated as long as the roots are signed, they could be provided by untrusted third parties, and not necessarily by the bank. By verifying the hash chain the merchant would detect if a database is corrupted. A large merchant could even have its own database.