Practical Non-Interactive Key Distribution Based on Pairings

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May 24, 2002

Abstract

We propose a practical non-interactive key distribution protocol based on pairings and define a notion of security for such a scheme. We prove the security of the system in this setting under the GDBH assumption, and present some possible realisations using Weil or Tate pairings on supersingular and ordinary elliptic curves.

Keywords: key distribution, non-interactive, identity based cryptography, pairings.

1 Introduction

A non-interactive key distribution protocol is a way to create a shared secret between two parties, henceforth called "Alice" and "Bob" as usual to avoid confusion. While interactive protocols like the classical Diffie–Hellman key exchange require some communication between Alice and Bob to establish the common secret, this is not the case for non-interactive systems, hence the name.

Without further communication, the only information Alice and Bob have on each other are their respective identities, so that non-interactive cryptography is necessarily identity based, a concept introduced by Shamir in [Sha85]. In such a system, Alice derives the shared secret from her private key and Bob's identity, which can be seen as his public key, and Bob does likewise. Public keys being fixed by the participants' identities, Alice is clearly unable to determine her private key by herself; otherwise, Bob would be able to deduce Alice's private key as well, since he possesses the very same information on Alice's identity as herself. Thus, the help of a trusted third party is needed, the *Private Key Generator (PKG)*, who possesses additional privileged information in the form of a master-key. The role of the PKG is precisely to derive private keys from public identities using the master-key and to issue these private keys to their legitimate holders. Hence, another way of seeing the information flow in a non-interactive system is that the synchronous communication between Alice and Bob is replaced by asynchronous communication with the PKG.

In [Sha85], Shamir proposes only an identity based signature scheme, leaving open among others the problem of key distribution. Maurer and Yacobi in [MY92] suggest the first non-interactive key distribution scheme, based on discrete logarithms in $(\mathbb{Z}/n\mathbb{Z})^\times$ with composite n. However, some version of the protocol is soon shown to be insecure [LL92]. Even with the improvements of [MY96] it can be broken by two colluding participants who with a high probability can retrieve the PKG's secret information, that is the factorisation of n [KM99]. In the unbroken version, the modulus m is chosen as the product of two primes p such that the maximal prime factor q of $p-1$ is of medium size. To determine a private key, the PKG computes discrete logarithms modulo the prime factors of the $p-1$, which by Pollard's ρ algorithm can be done with a complexity of $O(\sqrt{q})$. An attacker may also profit from the special structure of the primes and factor n by Pollard's $p-1$-method in time essentially $O(q)$. The relatively small difference between the complexities for creating a key and for breaking the system induce an impractically high computational load on the PKG (cf. [LL92]).

An alternative protocol, suggested by Hühnlein, Jacobson and Weber in [HJW00], uses non-maximal imaginary quadratic orders. The PKG has to solve discrete logarithm problems in the class group of an imaginary quadratic field and in a finite field, and the fastest algorithm for the class group step known to date has a subexponential complexity with exponent $1/2$. A potential attacker is assumed to have to factor the discriminant, which can also be done in subexponential time with exponent $1/2$ by the elliptic curve method. Hence, this scheme also requires that the PKG disposes of an enormous computing power, and the margin between instances not manageable by the PKG and instances vulnerable by attacks is very small. Furthermore, it is uncertain how well a choice of parameters falling into today's small margin of security will resist the exponential growth of computing power predicted by Moore's law.

In his diploma thesis [Küg98], Kügler develops a key distribution system based on the discrete logarithm problem in $(\mathbb{Z}/n\mathbb{Z})^\times$ for composite n, in which the PKG can compute private keys in polynomial time.

None of the above protocols come with a formal proof of security.

The Weil and Tate pairings on elliptic curves have originally been introduced into cryptology to break certain elliptic curve cryptosystems [MOV93, FR94]. Joux has recently shown in [Jou00] that these pairings also present a constructive facet. Numerous applications have since then emerged, ranging from identity based encryption [BF01] over interactive key agreement protocols [Sma01, ARP02] to short [BLS01] or identity based signatures [CC02, Hes].

In this article, we propose a non-interactive identity based key distribution protocol in the setting of a very general pairing, whose properties are reviewed in Section 3. The protocol itself is described in Section 4. The security of the scheme is based on the *Generalised Bilinear Diffie–Hellman Problem (GBDH)*, a natural generalisation of the BDH introduced in the long, online version of [BF01]. We define a notion of security and prove that the protocol is secure in the random oracle model assuming that the GBDH problem is hard, see Section 5. Concrete implementations are obtained, for instance, from the Tate or Weil pairings on supersingular or ordinary elliptic curves as described in Section 6. In this setting, the PKG can compute private keys in polynomial time by a scalar multiplication on the elliptic curve. The effort for an adversary to solve the GBDH problem, however, even when using the fastest algorithm known to date, is at least subexponential.

After having finished the present article, we found that the same protocol is described in [SOK], however, without a formal proof of security.

2 Remarks on non-interactive key distribution

The main motivation for non-interactive key distribution is to use the common secret existing between two parties as a secret key for a symmetric cipher. The result is an identity based encryption scheme.

One of the major drawbacks of identity based systems is the need for a Private Key Generator, allowing key escrow. A classical way to bypass this problem is to use a secret sharing scheme such as Shamir one's to split the master-key between several PKGs. However, in some cases, a natural authority exists that can serve as the PKG. For example if the system is to be used between employees of a company, that company can be the PKG. Other interesting applications include mobile telephone communication and network routing, where the network operators are natural PKGs.

One should beware that the amount of secret information shared between two parties is usually quite limited and that the master-key presents a very attractive target for a potential attacker, whence it may be important that the master-key be changed periodically. Of course, the PKG would then have to

keep all the used master-keys in memory, so that users could at any time obtain their private key corresponding to a past period from the PKG.

3 Pairings and the GBDH problem

In the remaining sections, we let $(G,+)$, $(\hat{G},+)$ and $(V,\times)$ denote groups of prime order ℓ. The sets of their non-neutral elements are denoted by G^*, $\hat{G}^*$ and V^*, respectively. We suppose that $e : G \times \hat{G} \to V$ is a pairing satisfying the following properties:

- **Bilinearity:** $e(aP, bQ) = e(P,Q)^{ab}$ for all $P \in G$, $Q \in \hat{G}$, a, $b \in \mathbb{Z}$.
- **Non-degeneracy:** there are $P \in G$ and $Q \in \hat{G}$ such that $e(P,Q) \neq 1$. In our setting of prime order groups this is equivalent to $e(P,Q) \neq 1$ for all $P \in G^*$, $Q \in \hat{G}^*$.
- **Computability:** given $P \in G$ and $Q \in \hat{G}$, $e(P,Q)$ can be efficiently computed.

For instance, the Tate and Weil pairings on elliptic curves have these properties, cf. Section 6.

The security of the key exchange protocol relies on the following problem, baptised the **Generalised Bilinear Diffie–Hellman Problem (GBDH)**: given (P,Q,aP,bQ,cP,cQ), compute $e(P,Q)^{abc}$. This is the same problem as the Bilinear Diffie–Hellman Problem introduced in the extended online version of [BF01], except that we allow the groups G and $\hat{G}$ to be different. A probabilistic algorithm $\mathcal{A}$ is said to (t,ε)-solve GBDH in $(G,\hat{G},V,e)$ if $\mathcal{A}$ runs in time at most t and correctly solves the problem with probability at least ε, that is,

$$\text{Prob}\left(\mathcal{A}\left(P,Q,aP,bQ,cP,cQ\right) = e(P,Q)^{abc}\right) \geq \varepsilon.$$

The probability is taken over the uniformly and independently distributed $P \in G$, $Q \in \hat{G}$ and a, b, $c \in \mathbb{F}_\ell^\times$ and over the random choices of $\mathcal{A}$.

4 The non-interactive key distribution protocol

The protocol can be naturally divided into four distinct algorithms: **Setup**, **Master-key generation**, **Private key distribution** and **Common secret computation**.

- **Setup:** choose G, $\hat{G}$, V and e as in Section 3, and let $H : \{0,1\}^* \to G$ and $\hat{H} : \{0,1\}^* \to \hat{G}$ be cryptographic hash functions. All these parameters are publicly known.
- **Master-key generation:** the PKG chooses a random master-key $s \in [1, \ell - 1]$.
- **Private key distribution:** whenever a user A first wishes to use the system, he contacts the PKG and asks for his private key pair. Using A's identity ID_A, the PKG computes A's private key pair $(S_A, \hat{S}_A) = (sH(\mathrm{ID}_A), s\hat{H}(\mathrm{ID}_A))$ and sends it to A.
- **Common secret computation:** suppose that users A and B wish to create a common secret key. A computes B's public key

$$(P_B, Q_B) = (H(\mathrm{ID}_B), \hat{H}(\mathrm{ID}_B))$$

and conversely B computes

$$(P_A, Q_A) = (H(\mathrm{ID}_A), \hat{H}(\mathrm{ID}_A)).$$

Then A can compute

$$(e(S_A, Q_B), e(P_B, \hat{S}_A)),$$

and B can compute

$$(e(P_A, \hat{S}_B), e(S_B, Q_A)).$$

The bilinearity of e makes it easy to see that the computed tuples are in fact equal and thus constitute a secret shared between A and B.

5 Security

5.1 Attack scenario

In the non-interactive cryptographic setting of the previous section, the only observable traffic is the distribution of private keys. It is thus natural to consider the protocol secure if the corruption of an arbitrary number of private keys does not reveal the shared secret between two further participants. In particular, a colluding group of participants who reveal their private keys to one another then does not gain any insight into other people's common secrets. Precisely, an adversary $\mathcal{A}$ is said to (t, ε)-break the protocol if it runs in time at most t and has advantage at least ε in the following game.

- **Setup:** the challenger publishes the general system parameters $(G, \hat{G}, V, \ell, e, H, \hat{H})$.
- **Extraction queries:** $\mathcal{A}$ issues a number of extraction queries ID_1, ID_2, ..., ID_n to the challenger, who, upon receiving the query ID_i, computes the tuple $(sH(\mathrm{ID}_i), s\hat{H}(\mathrm{ID}_i))$ and sends it back to $\mathcal{A}$.
- **Guess:** Once $\mathcal{A}$ decides that it has collected enough information, it picks two identities ID_A and ID_B, different from all the ID_i, and publishes a quadruple $(\mathrm{ID}_A, \mathrm{ID}_B, \alpha, \beta)$.

The attacker $\mathcal{A}$'s advantage is defined as:

$$\mathrm{Adv}(\mathcal{A}) = p_{\mathcal{A},1} + p_{\mathcal{A},2}$$

with

$$p_{\mathcal{A},1} = \mathrm{Prob}\left(e(H(\mathrm{ID}_A), \hat{H}(\mathrm{ID}_B))^s = \alpha\right)$$

and

$$p_{\mathcal{A},2} = \mathrm{Prob}\left(e(H(\mathrm{ID}_B), \hat{H}(\mathrm{ID}_A))^s = \beta\right).$$

5.2 Security proof

In this section, we show that the GBDH problem and the security of the non-interactive key distribution protocol of Section 4 are polynomially equivalent.

Proposition 1 *If the GBDH problem in some setting $(G, \hat{G}, V, \ell, e)$ can be (t, ε)-solved, then the key distribution protocol in the setting $(G, \hat{G}, V, \ell, e, H, \hat{H})$ can be $(t + \delta, \varepsilon)$-broken, where δ is the time needed to carry out two extraction queries and to compute one hash value of H and of $\hat{H}$.*

Proof: An attacker on the protocol may extract two key pairs (P, sP) and (Q, sQ) with randomly chosen $P \in G$ and $Q \in \hat{G}$. He randomly selects two identitites ID_A and ID_B and computes $R = H(\mathrm{ID}_A) = aP$ and $S = \hat{H}(\mathrm{ID}_B) = bQ$ with unknown, but random a and b. The solution to the GBDH instance (P, Q, R, S, sP, sQ) provides the attacker with the shared secret between A and B. □

Theorem 2 *Let the hash functions H and $\hat{H}$ be given by random oracles. Suppose that there is some adversary $\mathcal{A}$ who (t, ε)-breaks the protocol with parameters $(G, \hat{G}, V, \ell, e, H, \hat{H})$. Assume furthermore that an upper bound q_E on the number of extraction queries issued by $\mathcal{A}$ is known. Then there is an algorithm $\mathcal{B}$ that $(t', \varepsilon/(2\,e^2\,(1+q_E)^2))$-solves the GBDH problem for $(G, \hat{G}, V, \ell, e)$, where e is Euler's number,*

$$t' = Kt(t_1 + t_2 + \log q_E) + t_3,$$

K is a small constant and

t_1	*is the time needed to carry out a scalar multiplication in G or $\hat{G}$ or an exponentiation in V*
t_2	*is the time needed to generate a random bit*
$\log(q_E)$	*is the time needed to locate an entry in an ordered list with at most q_E entries*
t_3	*is the time required for the extended Euclidean algorithm on numbers in the interval $[1, \ell - 1]$.*

Notice that in general, t' will be t times some polynomial in $\log \ell$, and $\log \ell \leq t$ since $\mathcal{A}$'s output is an element of V, so that in fact t' is polynomial in t. The assumption that an upper bound q_E on the number of extraction queries of $\mathcal{A}$ or, *a forteriori*, on its running time $t \geq q_E$ be known by $\mathcal{B}$, certainly shows limitations of the theorem. However, it seems to be commonly adopted in the literature, cf. [BF01, BLS01].

Proof:

$\mathcal{B}$ has as input a random and uniformly distributed instance $(P, Q, P_a, Q_b, P_c, Q_c) = (P, Q, aP, bQ, cP, cQ)$ of the GBDH problem. For finding the solution $e(P,Q)^{abc}$ with $\mathcal{A}$'s assistance, $\mathcal{B}$ has control over the hash functions H and $\hat{H}$. Basically, when queried for a hash value of, say, H, it outputs a random group element, obtained as a random multiple of P or P_a. Thus $\mathcal{B}$ conforms to the random oracle model (to $\mathcal{A}$, the hash function appears as a random function) while at the same time keeping track of additional information (the discrete logarithms with respect to the bases P or P_a). Of course, as a is unknown to $\mathcal{B}$, it may control only one of the discrete logarithms. To be able to answer to extraction queries, $\mathcal{B}$ should attach multiples of P to the corresponding identities; to retrieve the solution to the GBDH problem, it should attach a multiple of P_a to the identity for which $\mathcal{A}$ finally emits its guess. These requirements put $\mathcal{B}$ into a dilemma, because $\mathcal{A}$ may request hash values *before* deciding to query the private key or to emit a guess for the corresponding identity. To solve the problem, $\mathcal{B}$ randomly goes for multiples of P or P_a and declares failure whenever it realises that it has made the wrong choice previously. The probabilities of selecting P or P_a must depend on q_E, since otherwise $\mathcal{B}$'s success probability becomes exponentially small for q_E tending to infinity. The more extraction queries $\mathcal{A}$ makes, the more often $\mathcal{B}$ has to return a multiple of P. This is the reason why $\mathcal{B}$ needs to know at least an upper bound on q_E, and furthermore its success probability decreases the more private keys $\mathcal{A}$ extracts. In detail, $\mathcal{B}$ implements the following routines:

H queries: $\mathcal{B}$ keeps an initially empty list L of tuples $(X, R, h, u) \in \{0,1\}^* \times G \times [1, \ell - 1] \times \{0, 1\}$, sorted according to X. When $\mathcal{A}$ queries for the hash value of some bit string X, $\mathcal{B}$ checks if L contains a tuple (X, R, h, u). If this is not the case, then $\mathcal{B}$

- picks uniformly a random $h \in [1, \ell - 1]$
- picks $u \in \{0, 1\}$ with $\mathrm{Prob}(u = 0) = \delta$, where δ is a parameter to be determined later
- if $u = 0$, sets $R = hP$, otherwise sets $R = hP_a$
- appends (X, R, h, u) to L

Finally, it sends R to $\mathcal{A}$.

$\hat{H}$ queries: These are handled in the same way, $\mathcal{B}$ keeping a list $\hat{L}$ and returning a multiple of Q with probability δ and a multiple of Q_b with probability $1 - \delta$.

Extraction queries: To answer to a query issued by $\mathcal{A}$ upon the string ID, the algorithm $\mathcal{B}$:

- queries H and $\hat{H}$ as described above to make sure that L contains a tuple of the form (ID, R, h, u) and $\hat{L}$ a tuple of the form $(\mathrm{ID}, S, \hat{h}, \hat{u})$
- checks if $u = 1$ or $\hat{u} = 1$, in which case it reports failure

- computes the tuple $(hP_c, \hat{h}Q_c)$ and sends it to $\mathcal{A}$

Guess: Upon receiving the guess $(\mathrm{ID}_A, \mathrm{ID}_B, \alpha, \beta)$ from $\mathcal{A}$, the algorithm $\mathcal{B}$

- proceeds as in the case of H and $\hat{H}$ queries to make sure that L contains tuples of the form $(\mathrm{ID}_i, R_i, h_i, u_i)$ and $\hat{L}$ tuples of the form $(\mathrm{ID}_i, S_i, \hat{h}_i, \hat{u}_i)$ for $i = A, B$
- uniformly picks a random $t \in \{0,1\}$
- if $t = 0$, checks if $u_A = 1$ and $\hat{u}_B = 1$ (otherwise reports failure), then outputs $\alpha^{1/(h_A \hat{h}_B)}$ as a guess
- if $t = 1$, checks if $u_B = 1$ and $\hat{u}_A = 1$ (otherwise reports failure), then outputs $\beta^{1/(h_B \hat{h}_A)}$ as a guess

Now, suppose that $\mathcal{B}$ does not abort and let γ be its output. With probability $1/2$, we have $t = 0$, whence $u_A = 1$, $\hat{u}_B = 1$, $H(\mathrm{ID}_A) = h_A P_a = a\,h_A\,P$, $\hat{H}(\mathrm{ID}_B) = \hat{h}_B Q_b = b\,\hat{h}_B\,Q$ and $\gamma = \alpha^{1/(h_A \hat{h}_B)}$. Independently, with probability $p_{\mathcal{A},1}$, we have $\alpha = e(H(\mathrm{ID}_A), \hat{H}(\mathrm{ID}_B))^c$. Thus, the following event happens with probability $p_{\mathcal{A},1}$:

$$\begin{aligned}\gamma &= \alpha^{1/(h_A \hat{h}_B)} = e\left(H(\mathrm{ID}_A), \hat{H}(\mathrm{ID}_B)\right)^{c/(h_A \hat{h}_B)} \\ &= e\left(a\,h_A\,P, b\,\hat{h}_B\,Q\right)^{c/(h_A \hat{h}_B)} = e(P,Q)^{abc},\end{aligned}$$

where the last equality follows from the bilinearity of the pairing.

A similar analysis for $t = 1$ shows that $\mathcal{B}$ guesses correctly with an additional probability of $p_{\mathcal{A},2}/2$. Since these two events are disjoint, $\mathcal{B}$'s guess is correct with a total probability of $(p_{\mathcal{A},1} + p_{\mathcal{A},2})/2 \geq \varepsilon/2$ whenever it does not abort.

We now compute the probability for $\mathcal{B}$ to abort. Let q_E be the number of extraction queries issued by $\mathcal{A}$. Then the probability of non-abortion during each extraction query being δ^2 and the probability of non-abortion during the guess phase being $(1-\delta)^2$, the overall probability of non-abortion is at least (as q_E has been taken to be an upper bound on the actual number of extraction queries) $\delta^{2q_E}(1-\delta)^2$. Minimising this function, we find the optimal value $\delta = q_E/(1+q_E)$ and an overall probability of non-abortion of at least $1/(e(1+q_E))^2$. Hence, the probability that $\mathcal{B}$ outputs the correct solution to the GBDH instance is at least $\varepsilon/\left(2e^2(1+q_E)^2\right)$.

The running time analysis of $\mathcal{B}$ is straightforward except for the computation of the root in V. Notice that for an element α in an arbitrary group of order ℓ and $r \in [1, \ell-1]$, the root $\alpha^{1/r}$ can be obtained by computing $s = r^{-1} \bmod \ell$ *via* the extended Euclid algorithm and raising α to the power s. $\square$

Proposition 1 and Theorem 2 show that the GBDH problem and the key distribution protocol are polynomially equivalent, and show accurately how the running times and success probabilities are transformed during the reductions. Assuming that the GBDH problem is hard, the security of the protocol is thus established.

It is possible to furthermore formalise the security notion from a complexity theoretic point of view. To do so, it is necessary to introduce infinite families of problem instances. Let thus $\mathcal{F} = \Big((G_k, \hat{G}_k, V_k, \ell_k, e_k)\Big)_{k\in\mathbb{N}}$ be a family of GBDH parameters as above. We say that $\mathcal{F}$ satisfies the polynomial GBDH assumption if, for any polynomials P and Q in $\mathbb{Z}[X]$, there is no randomised algorithm $\mathcal{A}$ that $(P(k), 1/Q(k))$-solves the GBDH problem for $(G_k, \hat{G}_k, V_k, \ell_k, e_k)$ for all $k \in \mathbb{N}$. The above proof shows that under the random oracle model, if $\mathcal{F}$ satisfies the polynomial GBDH assumption, then the protocol with parameters from $\mathcal{F}$ is secure in the sense that no polynomial time algorithm achieves a polynomial advantage in breaking the protocol.

Similarly, one might admit adversaries with subexponential computing power and define in the same way the subexponential GBDH assumption. Then our security analysis shows that under the subexponential GBDH assumption, no algorithm of subexponential complexity can break the protocol with a subexponential advantage.

6 Implementation using elliptic curves

6.1 The Weil and Tate pairings on elliptic curves

In this section we summarise the properties of pairings on elliptic curves. More detailed descriptions can be found in [Sil86, Men93, Eng99]. Let $E(\mathbb{F}_q)$ be an elliptic curve of order m defined over $\mathbb{F}_q$, and let ℓ be a prime factor of m. Moreover, supposing that ℓ does not divide $q-1$, we define k to be the smallest integer such that

$$\ell \mid q^k - 1$$

(k is often referred to as the *MOV degree*).

Let $E[\ell]$ denote the set of ℓ-torsion points of E over some closure of $\mathbb{F}_q$. Theorem 1 of [BK98] then states that $E[\ell] \subseteq E(\mathbb{F}_{q^k})$. The Weil and Tate pairings on $E[\ell]$ are maps

$$e : E[\ell] \times E[\ell] \to \mathbb{F}_{q^k}^{\times}$$

which satisfy the properties of bilinearity and non-degeneracy stated above. Moreover, if the MOV degree k is not too large, then the Weil and Tate pairings are efficiently computable using an algorithm due to Miller [Mil86].

Note that the non-degeneracy property implies that if P and Q are ℓ-torsion points, then $e(P, Q) \neq 1$ if and only if these points are linearly independent since $E[\ell]$ is isomorphic to $\mathbb{Z}/\ell\mathbb{Z} \times \mathbb{Z}/\ell\mathbb{Z}$.

6.2 Description of the protocol using supersingular elliptic curves

A remarkable fact about supersingular elliptic curves is that their MOV degree is always inferior to 6 [MOV93], hence the Weil and Tate pairings are always efficiently computable. Moreover, when working with supersingular curves, it is often possible to find an efficiently computable injective endomorphism $\phi : E(\mathbb{F}_q)[\ell] \to E(\mathbb{F}_{q^k})[\ell]$ which is rational over $\mathbb{F}_q$. One may then use a modified Weil pairing $\hat{e} : E(\mathbb{F}_q)[\ell] \times E(\mathbb{F}_q)[\ell] \to \mathbb{F}_{q^k}^{\times}$, obtained from the original Weil pairing e by $\hat{e}(P, Q) = e(P, \phi(Q))$. This pairing clearly satisfies the bilinearity and non-degeneracy properties and is efficiently computable. Examples of such curves with associated morphisms can be found in [BF01, BLS01, Hes].

The protocol described in Section 4 can now be implemented using the pairing $\hat{e}$ with $G = \hat{G} = (E/\mathbb{F}_q)[\ell]$. In this case, only one hash function $H : \{0,1\}^* \to G$ is needed. An extension to higher genus supersingular curves is also straightforward.

6.3 Description of the protocol using ordinary elliptic curves

Suppose that we have an ordinary elliptic curve E defined over $\mathbb{F}_q$, such that $m = \#E(\mathbb{F}_q)$ has a large prime factor ℓ and the MOV degree of $E(\mathbb{F}_q)$ with respect to ℓ is small.

This implies that the Weil and Tate pairings can be efficiently computed on $E[\ell]$, and the protocol described in Section 4 can be directly implemented, letting $G = E(\mathbb{F}_q)[\ell]$, $\hat{G} = E[\ell] \setminus G^*$ and $V = \mathbb{F}_{q^k}$, and using either the Weil or the Tate pairing.

In [DEM02] it is shown how to generate ordinary curves over finite prime fields having a specified MOV degree. For example, the curve $E : y^2 = x^3+ax+b$ defined over $\mathbb{F}_p$, with

$$
\begin{aligned}
p &= 845374210422870575471023560981263755113163526494385544386734375804852490290377727350819\backslash\\
&\quad 81471764171246449562935954732095525771726428 70167 \quad (451\text{bits})\\
\ell &= 22986058416228970361863730695421846214124773102557372485666471903661 \quad (223\text{bits})\\
a &= 6822037327990046413951088439860872817180083404468443210433999681670244654089761251331820\backslash\\
&\quad 0933819915914342234482509839950285379142727253 1\\
b &= 5091926477406250722887108016557836789437239556470766218100447813239589852331179508280006\backslash\\
&\quad 0801861362386930599139488190681937050289718651 59
\end{aligned}
$$

has MOV degree $k = 10$ and contains a subgroup of prime order ℓ.

7 Conclusion

We have presented a practical non-interactive key distribution protocol based on pairings and defined a notion of security for such a scheme. This cryptosystem satisfies this notion of security in the random oracle model if the GBDH assumption holds. In particular, the protocol is secure against an arbitrary number of colluding attackers.

We have proposed realisations of the protocol using Weil or Tate pairings on supersingular or ordinary elliptic curves.

Used together with a symmetric cipher such as the AES, the scheme achieves identity based encryption.

Recently, the concept of hierarchical identity based system has been defined, and such schemes have been proposed [HL02, GS]. Using the same ideas, it is easy to see that the protocol can also be transformed into a hierarchical system.

Acknowledgements: We thank François Morain for valuable discussions concerning this work. The second author gratefully acknowledges being supported by a fellowship within the postdoctoral programme of the German Academic Exchange Service (DAAD). This research was partially supported by the French Ministry of Research — ACI Cryptologie.

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